



## 8. STRUCTURAL ANALYSIS

The addition of a green roof to the SLCC imposes additional gravity loads on the structure. The conclusion to include an extensive green roof imposes a minimum superimposed dead load of 25 pounds per square foot (DC Greenworks). This section evaluates the current roof deck and support system's capacity to carry this additional gravity load.

### 8.1. EXISTING CONDITIONS

The SLCC has three roof levels: a two (2) story wing roof; a three (3) story wing roof; and an atrium roof. The proposed green roof will be applied to the first two roof surfaces which cover the majority of the building footprint. These roofs are composed to two typical constructions. The predominant roof surface is designed to be unoccupied and consists of 20 GA wide rib steel roof deck, 3" rigid insulation, and a waterproof membrane (Figure 8.1). This roof is supported by K-shape open-web steel joists and W-shape girders. The other typical roof is located exclusively on the third floor roof around the rooftop mechanical equipment and is intended to carry semi-frequent occupant loads. This roof is constructed with 18GA roof deck rather than 20GA deck. This construction is supported by W-shape steel beams and girders. The load path for both roof types leads from the girders to W-shape steel columns and directly down to the foundation.

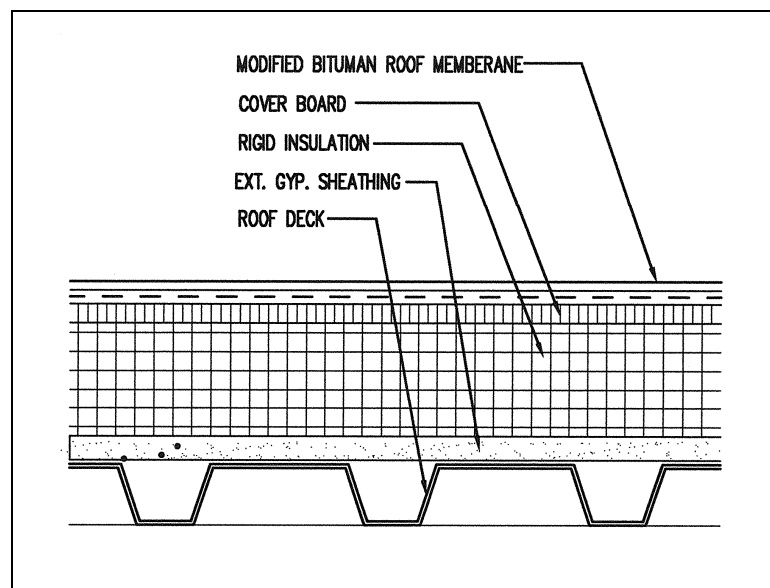


Figure 8.1: Typical roof construction detail.



## 8.2. STRUCTURAL ANALYSIS METHODOLOGY

Design roof loads are determined using the structural cover sheet of the SLCC Construction Documents and Table C3-1 from ASCE 7-05. The corrected snow load for the roof level is derived from the contract documents. The additional extensive green roof dead load is given by DC Greenworks. These loads are combined to determine the total dead load for each roof design. Dead and live loads were added together to determine total gravity loads. See Table 8.1 for each of these loads.

Structural Roof Loads		
Roof Dead Load		
Construction	Material	PSF
Green Roof	Soil, plants, etc.	25.0
Waterproof Membrane	Smooth, bituminous membrane	1.5
Insulation	Rigid insulation	1.0
Roof Deck	20G - 18G Steel, 1 1/2" deep	3.0
MEP	Mech, Elec. equipment	5.0
Ceiling	Ceiling panels, fasteners	2.0
Collateral		5.0
<b>TOTAL</b>	Original Roof Design	<b>17.5</b>
	Green Roof Design	<b>42.5</b>
Roof Live Load		
Category		PSF
Ground Snow Load		30.0
Flat Roof Snow Load (Governs)		23.0
People		20.0
<b>TOTAL</b>		<b>23.0</b>
TOTAL		PSF
Original Roof Design		<b>35.5</b>
Green Roof Design		<b>60.5</b>

Table 8.1: Expected gravity loads on roof.

Several members are checked for their capacity to carry the new green roof loads with hand calculations. These calculations find the maximum shear force, maximum moment, maximum allowable deflection, moment of inertia, and plastic section modulus. The results are then compared to the W-shape beam properties in AISC Steel Manual Table 3-6. Open-web steel joists are evaluated based on their capacity to carry maximum and total and live shear loads according to Steel Joist Institute Standard Load Tables. Girders are checked by their maximum shear force, maximum moment force, and plastic section modulus. See the sample calculations below for an example of this process.

A RAM Steel Model of the roof structure and top tier of columns include input based on the loads in Table 8.1 and physical dimensions of the actual building. The program computes loads for all joists, girders and columns and produces an output report suggesting sizes for these members.



### 8.2.1. STRUCTURAL EQUATIONS

Solve for:	Equation	[Units]
Deflection	$\Delta = (5 w l^4) / (384 E I_x)$	[in]
Maximum Deflection (total load)	$\Delta_{\max, \text{total}} = l / 240$	[in]
Maximum Deflection (live load)	$\Delta_{\max, \text{live}} = l / 360$	[in]
Maximum Service Load Moment	$M_{\max} = (w l^2) / 8$	[kip ft]
Maximum Service Load Shear Force	$V_{\max} = (w l) / 2 \leq V_n / \Omega_v$	[kip]
Plastic Section Modulus about x-axis	$Z_x \geq M_{\max} / F_y$	[in <sup>3</sup> ]

Variable	Symbol	[Units]
Uniformly Distributed Load	$w$	[kips/ft]
Span Length	$l$	[ft, in]
Modulus of Elasticity of Steel	$E = 29000$	[ksi]
Moment of Inertia of Cross Section	$I_x$	[in <sup>4</sup> ]
Maximum Shear Strength	$V_n$	[kips]
ASD Safety Factor	$\Omega_v = 1.67$	-
Specified Minimum Yield Stress (A992 Steel)	$F_y = 50$	[ksi]

### 8.2.2. ASSUMPTIONS:

- Member connections are sized based on designed capacity of members and future loads.
- If all members are sufficiently sized for the roof structure and its supporting columns, the supporting columns and caissons are also able to support the additional green roof load.



### 8.2.3. FREE BODY DIAGRAMS

The figures below depict the typical load patterns for the structural elements analyzed in this thesis with hand calculations. Figure 8.2 shows the plans for the two typical bays, Figure 8.3 is a free body diagram of the loading pattern of a typical girder, and Figure 8.4 presents the loading pattern for a typical joist.

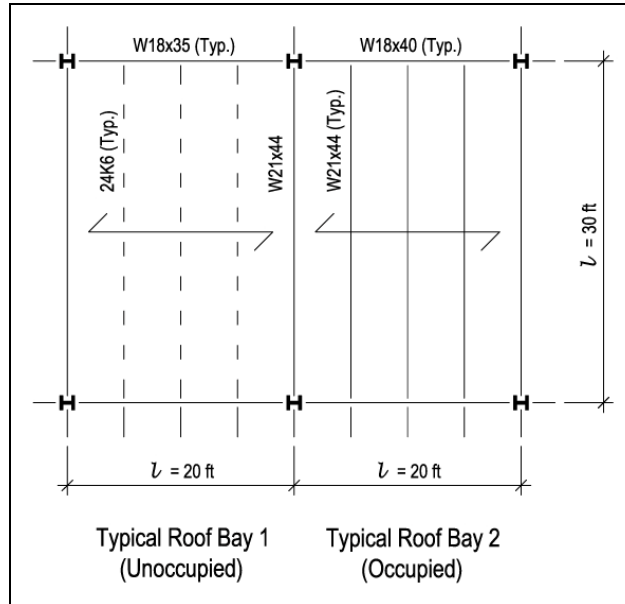


Figure 8.2: Plans of typical structural bays studied.

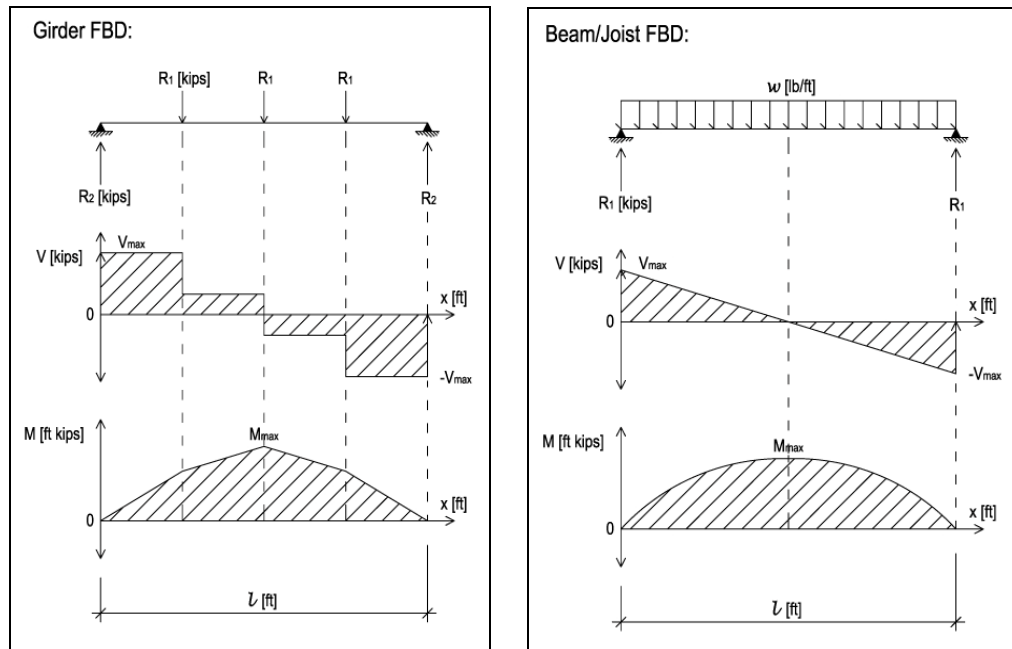


Figure 8.3: Free body diagram of a typical girder.

Figure 8.4: Free body diagram of a typical joist.



### 8.3. SAMPLE CALCULATIONS

#### 8.3.1. SAMPLE JOIST CALCULATION

This is the calculation for the typical bay 2 (18GA deck) (Figure 8.2) green roof loading case according to the typical joist loading pattern (Figure 8.4).

$$\begin{aligned}\Delta_{\text{total}} &= (5 w L^4) / (384 E I_x) \\ &= (5 (0.3575)(30)^4(12)^3) / (384 (29000) I_x) \\ &= 224.67 \text{ in}^5 / I_x\end{aligned}$$

$$\begin{aligned}\Delta_{\text{max, total}} &= L / 240 \\ &= (30 * 12) / 240 \\ &= 1.5 \text{ in}\end{aligned}$$

$$\begin{aligned}\Delta_{\text{total}} &\leq \Delta_{\text{max, total}} \\ 224.67 \text{ in}^5 / I_x &\leq 1.5 \text{ in}\end{aligned}$$

$$\rightarrow I_x \geq 149.78 \text{ in}^4 \quad (\text{GOVERNS})$$

(See AISC Steel Const. Manual Table 1-1)

$$\begin{aligned}\Delta_{\text{live}} &= (5 (0.215)(30)^4(12)^3) / (384 (29000) I_x) \\ &= 135.12 \text{ in}^5 / I_x\end{aligned}$$

$$\begin{aligned}\Delta_{\text{max, live}} &= L / 360 \\ &= (30 * 12) / 360 \\ &= 1.0 \text{ in}\end{aligned}$$

$$\begin{aligned}\Delta_{\text{live}} &\leq \Delta_{\text{max, live}} \\ 135.12 \text{ in}^5 / I_x &\leq 1.0 \text{ in}\end{aligned}$$

$$\begin{aligned}\rightarrow I_x &\geq 135.12 \text{ in}^4 \\ 135.12 \text{ in}^4 &< 149.78 \text{ in}^4 \quad (\text{DOES NOT GOVERN})\end{aligned}$$

$$\begin{aligned}V_{\text{max}} &= (w L) / 2 \\ &= (.3575)(30) / 2\end{aligned}$$

$$V_{\text{max}} = 5.36 \text{ kips} \quad (\text{See AISC Steel Const. Manual Table 3-6})$$

$$\begin{aligned}M_{\text{max}} &= (w L^2) / 8 \\ &= (.3575)(30^2) / 8 \\ &= 40.22 \text{ ft kips}\end{aligned}$$

$$\begin{aligned}Z_x &\geq M_{\text{max}} / F_y \\ &\geq (40.22)(12) / 50\end{aligned}$$

$$Z_x \geq 9.84 \text{ in}^3 \quad (\text{See AISC Steel Const. Manual Table 3-6})$$

→ Select a **W12x22** Member ( $I_x = 156 \text{ in}^4$ ,  $V_{\text{max}} = 64 \text{ kips}$ ,  $Z_x = 29.3 \text{ in}^3$ )



Check:  $\Delta_{\text{live}} = 135.12\text{in}^5 / 156\text{in}^4$   
 $= 0.86 \text{ in} \leq 1.0 \text{ in}$  **OK**

$\Delta_{\text{total}} = 224.67\text{in}^5 / 156\text{in}^4$   
 $= 1.44 \text{ in} \leq 1.5 \text{ in}$  **OK**

### 8.3.2. SAMPLE GIRDER CALCULATION

This is the calculation for a girder between typical bay 1 and 2 for the green roof loading case according to the typical girder loading pattern (Figure 8.3).

$$V_{\text{max}} = \sum R_i / 2$$
$$= (4.01 + 6.02)(3) / 2$$

$$V_{\text{max}} = \mathbf{15.04 \text{ kips} + 0.5 * \text{Self Weight}}$$

$$M_{\text{max}} = \sum \text{Areas under half of shear curve}$$
$$= (5)(5.02 + 15.04)$$
$$= 100.28 \text{ ft kips}$$

$$Z_x \geq M_{\text{max}} / F_y$$
$$\geq (100.28)(12) / 50$$
$$Z_x \geq \mathbf{24.07 \text{ in}^3}$$

→ Select a **W12x19** Member ( $V_{\text{max}} = 57.2 \text{ kips}$ ,  $Z_x = 24.7 \text{ in}^3$ )



#### 8.4. EXISTING STRUCTURE EVALUATION

The results of the hand calculations in Table 8.2 and Table 8.3 indicate that the selected typical members have the capacity to carry the additional gravity load of the green roof.

#### Joist/Beam Selections for Typical Bays<sup>1</sup>

Bay	Roof Type	Member Selection <sup>2</sup>	Actual Member	Comments
Typical Bay No. 1	Original	20K4	24K6	3 rows bridging
	Green	20K4	24K6	<b>Original Design OK</b>
Typical Bay No. 2	Original	W12x19	W21x44	
	Green	W12x22	W21x44	<b>Original Design OK</b>

<sup>1</sup> N.B. Span = 30 ft, 24" deep structural plenum.

<sup>2</sup> Assume L/240 Max. Deflection

**Table 8.2:** Joist and beam selections for original, green roofs.

#### Girder Selections for Typical Bays<sup>1</sup>

Bay	Roof Type	Member Selection	Actual Member	Comments
Typical Bay No. 1	Original	W12x16	W18x40	
	Green	W12x19	W18x40	<b>Original Design OK</b>
Typical Bay No. 2	Original	W12x16	W24x84	
	Green	W12x19	W24x84	<b>Original Design OK</b>

<sup>1</sup> N.B. Span = 20 ft, 24" deep structural plenum.

**Table 8.3:** Girder selections for original, green roofs.



A model of the roof structure and supporting columns for one floor height below the roof was produced in RAM Steel (Figure 8.5, Figure 8.6). Both the original and green roof loading cases were analyzed and all beams, joists, girders, and columns are found to be sufficient to carry both load cases. A full check of each member can be found in Appendix B and shows that every roof structure member is sufficient for the supplemental green roof load.

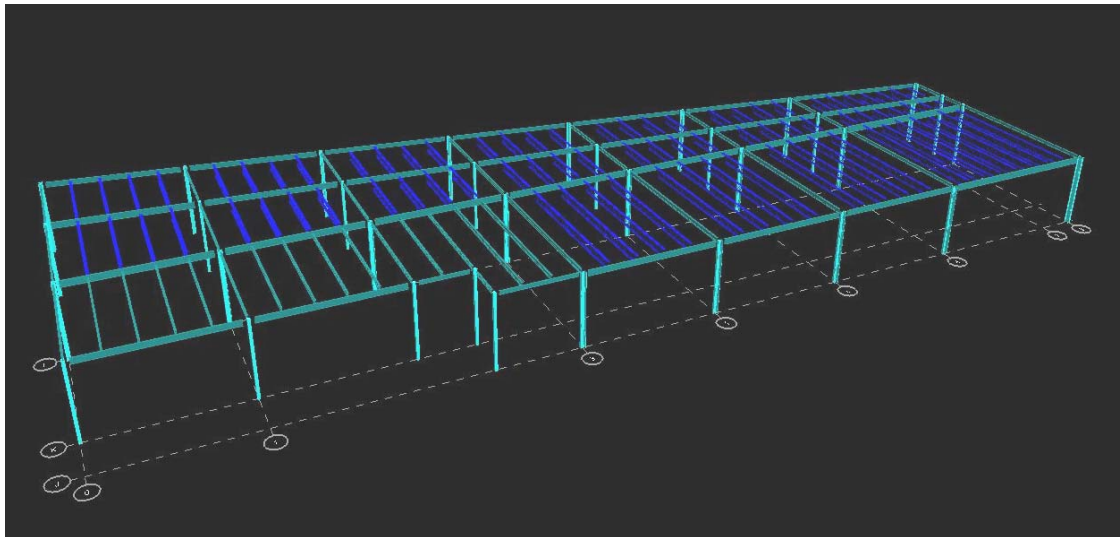


Figure 8.5: RAM Model of second floor roof.

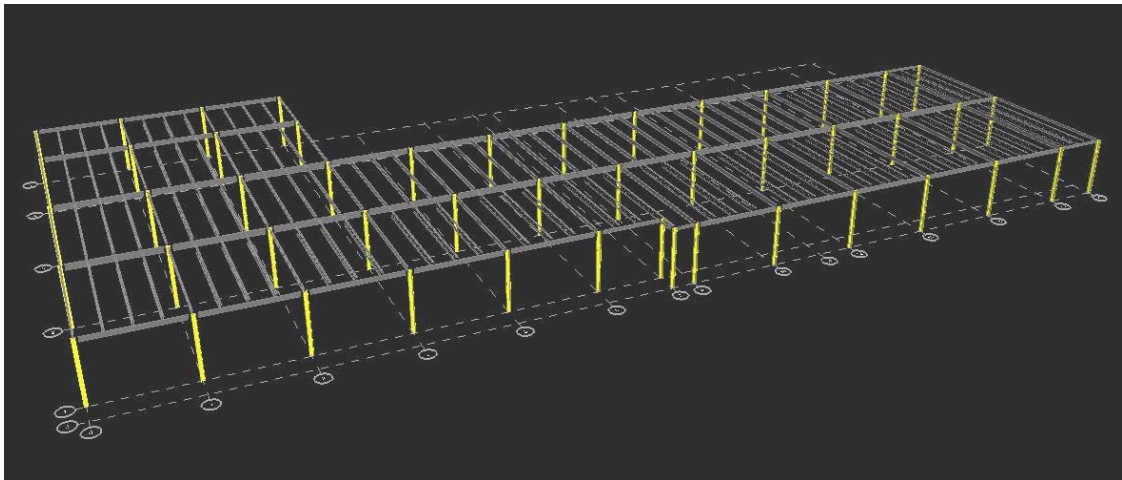


Figure 8.6: RAM Model of third floor roof.

## 8.5. CONCLUSION

The results of this structural analysis show that the originally designed structure should be capable of carrying the additional 25psf load of an extensive green roof. The structure is significantly oversized for the expected load cases. This is likely the product of using standard member sizes (e.g. W24 beams and K6 joists), safety factors, and allowances for future loads. Therefore, no changes to the structure are necessary for the proposed green roof.